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EJECTION OF PARTICLES FROM A CHARGED INFINITE STRIP.(U)

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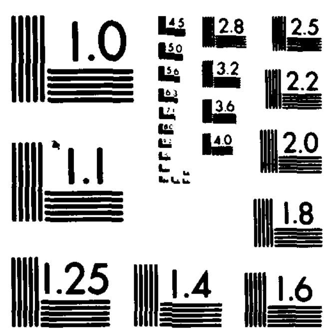

MICROCOPY RESOLUTION TEST CHART

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Trajectories of charged particles emitted from an infinite strip at a fixed potential with respect to and embedded in a unipotential ambient plane are calculated. Separation curves that divide emitted particles into those that escape from the emission plane and those that return to it are obtained. Results are compared with trajectories calculated in a previously studied, simpler model. Asymptotic properties are examined. The case of particles emitted from an embedded circular disk is discussed.		

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Preface

We thank H. A. Cohen for supporting this work.

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Illustrations

Ejection of Particles From a Charged Infinite Strip

1. INTRODUCTION

The problem of the emission of particles from separated electrified half planes has been considered by Shima, Kalman, and Carini.[1] The electrode geometry they treated is perhaps the simplest model one can use to approximate the emission of charged particles from an electrode at a given potential, towards a second electrode at another potential, fixed with respect to the first.

In this report we consider the next logical step in modeling, namely an infinite strip, of width bc (Figure 1) imbedded between two infinite half planes, ab, cd, placed symmetrically about the origin. We set the width bc equal to two, and the potential difference between interior and exterior electrodes equal to unity. As will subsequently be shown, there is a difference in asymptotic properties between the infinite half planes treated in the earlier model and the infinite strip treated here. Because of this, the behavior of particles ejected from the plane of the electrodes differs not only quantitively, but qualitatively as well, in these two cases. In particular, for the half plane case, every particle ejected from one electrode, with the electric field in the positive y direction above it, which crosses to the space over the other electrode with the electric field directed in the negative y direction,

(Received for publication 29 January 1982)

1. Shima, Y., Kalman, G., and Carini, P. (1981) Particle Trajectories In A Model Electric Field I, AFGL-TR-81-0167, ~~AD A104507~~.

ultimately returns to the plane of ejection. For the infinite strip case, particles ejected from the center of the strip generally escape and do not return to the ejection plane. This result seems to be largely independent of both angle and energy of ejection. As the point of emission approaches the edge of the strip, particles are returned to the ejection plane for a wider and wider set of initial angles and energies; and in the limit of an infinitesimal separation between the point of ejection and the strip edge, the results of Shima, Kalman, and Carini are, as would be expected, recovered.

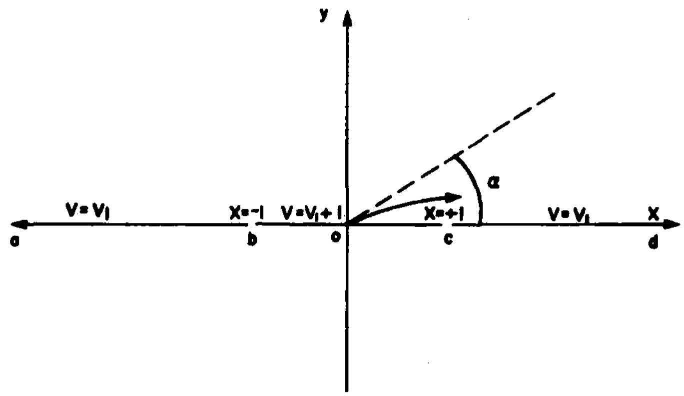

Figure 1. Geometry and Coordinate System for Emission From a Strip

2. THE EQUATIONS

The potential above an infinite strip of width 2, and with unit potential difference between strip and surrounding plane may be obtained[2] from the conformal transformation

$$w = \ln\left(\frac{z-1}{z+1}\right)$$

by taking the imaginary component of w and is found to be

$$V = V_1 + 1/\pi \tan^{-1}\left(\frac{2y}{x^2+y^2-1}\right). \quad (1)$$

2. Churchill, R. V. (1948) Complex Variables and Applications, McGraw-Hill, New York, Appendix II, Figure 19.

Here, $0 \leq \tan^{-1} \leq \pi$, V_1, is the potential of the plane outside the strip, and $V_1 + 1$ the potential inside, on the strip.

Taking derivatives, we have for the equations of motion

$$m\ddot{x} = e/\pi \; \frac{4xy}{(x^2 + y^2 - 1)^2 + 4y^2} \tag{2}$$

$$m\ddot{y} = e/\pi \; \frac{-2(x^2 - y^2 - 1)}{(x^2 + y^2 - 1)^2 + 4y^2} \tag{3}$$

or in non-dimensionalized form,

$$\frac{d^2x}{d\tau^2} = 1/\pi \; \frac{4xy}{(x^2 + y^2 - 1)^2 + 4y^2} \tag{4}$$

$$\frac{d^2y}{d\tau^2} = 1/\pi \; \frac{-2(x^2 - y^2 - 1)}{(x^2 + y^2 - 1)^2 + 4y^2} \; . \tag{5}$$

Here m and e are the particle mass and charge respectively, the dot is a time derivative, and

$$\tau = \sqrt{e/m}\, t \, .$$

If we set

$$x = 1 + \xi$$

$$y = \zeta$$

where ξ and ζ are small quantities, we approach the (positive) strip edge, and Eqs. (4) and (5) become

$$\frac{d^2\xi}{d\tau^2} = 1/\pi \; \frac{\zeta}{\xi^2 + \zeta^2} \tag{6}$$

$$\frac{d^2\zeta}{d\tau^2} = 1/\pi \; \frac{-\xi}{\xi^2 + \zeta^2} \tag{7}$$

which aside from the factor $1/\pi$ and reversals in signs are the equations of Shima, Kalman, and Carini.

3. TRAJECTORIES AND SEPARATION CURVES

We now solve Eqs. (4) and (5) numerically, using a Haming method[3] for the initial conditions

$$\tau = 0; \; x = x_o, \; y = 0, \; \frac{dx}{d\tau} = 2\, U_o \cos\alpha, \; \frac{dy}{d\tau} = 2\, U_o \sin\alpha \tag{8}$$

where U_o is the initial particle energy and α the angle of emission (see Figure 1). We do this for four initial coordinates,

$x_o = 0.0$	$(1 - x_o = 1.)$	$y_o = 0$
$x_o = 0.68$	$(1 - x_o \cong 0.3)$,	$y_o = 0$
$x_o = 0.90$	$(1 - x_o = 0.1)$,	$y_o = 0$
$x_o = 0.968$	$(1 - x_o \cong 0.03)$,	$y_o = 0$

for three initial energies,

$U_o = 0.01$, $\quad U_o = 1.0$, $\quad U_o = 100$;

and for a variety of initial angles α.

The results are shown in Figures 2, 3, 4, and 5. It is seen that for the most part the graphs terminate in straight lines with slopes of 45° on the log-log plot, indicating asymptotic motion in a force free field and an ultimate escape from the emission plane. It is only for $U_o \cong 1$, for lower emission angles, and for emission from near the strip edge that particles are returned to the plane of the strip.

In Figure 6 we see several trajectories for $U_o = 1$ and $1 - x_o = 1.0 \times 10^{-5}$. Since not only $1 - x_o$ but also the maximum value of $1 - x$ and the maximum value of y are much less than unity, we expect this case to revert to that studied by Shima, Kalman, and Carini, and the trajectories in the Figures are quite similar in form to those that they obtain.

Although the graphs shown in Figures 2 through 5 are useful as illustrations of the forms of typical trajectories, they contain more information on the specific trajectories shown than is likely to be of general interest. As a means of presenting

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3. Carnahan, B., Luther, H. A., and Wilkes, J. O. (1969) Applied Numerical Methods, John Wiley, New York, p 393 ff.

essential information on a wider variety of initial conditions, graphs of complete trajectories are highly inefficient. A more condensed form of information display would be a series of graphs giving the x-coordinate of the point of impact (for those particles which do return) as a function of the three initial parameters, x_o, U_o, α. Although such graphs are easy to obtain, they still require substantial amounts of computer time, and we do not present any graphs of this intermediate type here. A display which contains even less information on specific trajectories, but is most compact in presentation is a series of "separation curves", each one characterized by a single initial coordinate, x_o. With reference to that initial coordinate, any point (U_o, α) which falls above the designated curve represents an initial condition corresponding to a particle which ultimately escapes from the plane of ejection ($y = 0$). Conversely, any point falling below the designated curve represents an initial condition leading to a trajectory which ultimately returns to the ejection plane. From this single figure then (Figure 7) the behavior of particles with regard to ultimate emission or return, and for all initial conditions x_o, U_o, α can be inferred. For $x_o \lesssim 0.7$, all particles escape regardless of initial energy or initial angle. For particles $0.7 < x_o < 0.97$ only particles with low angles and a limited energy range U_o falling roughly between 0.1 and ~10 are returned. As x_o approaches the edge of the inner strip ($x_o > 0.97$) particles of increasingly higher initial angles and broader energy ranges are returned to the plane of origin.

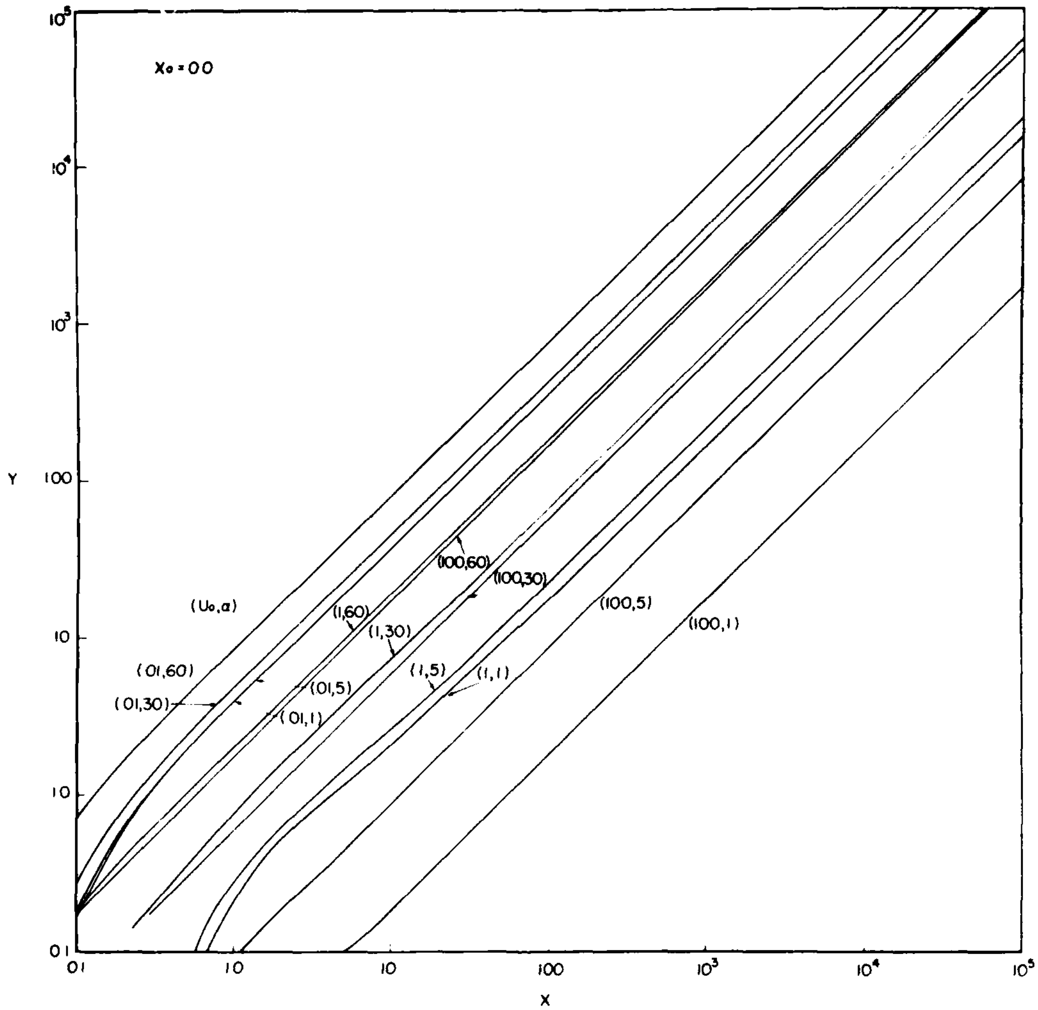

Figure 2. Typical Trajectories for $y_o = 0$, $x_o = 0.0$. The numbers in parenthesis are emission energy, U_o, and emission angle, α

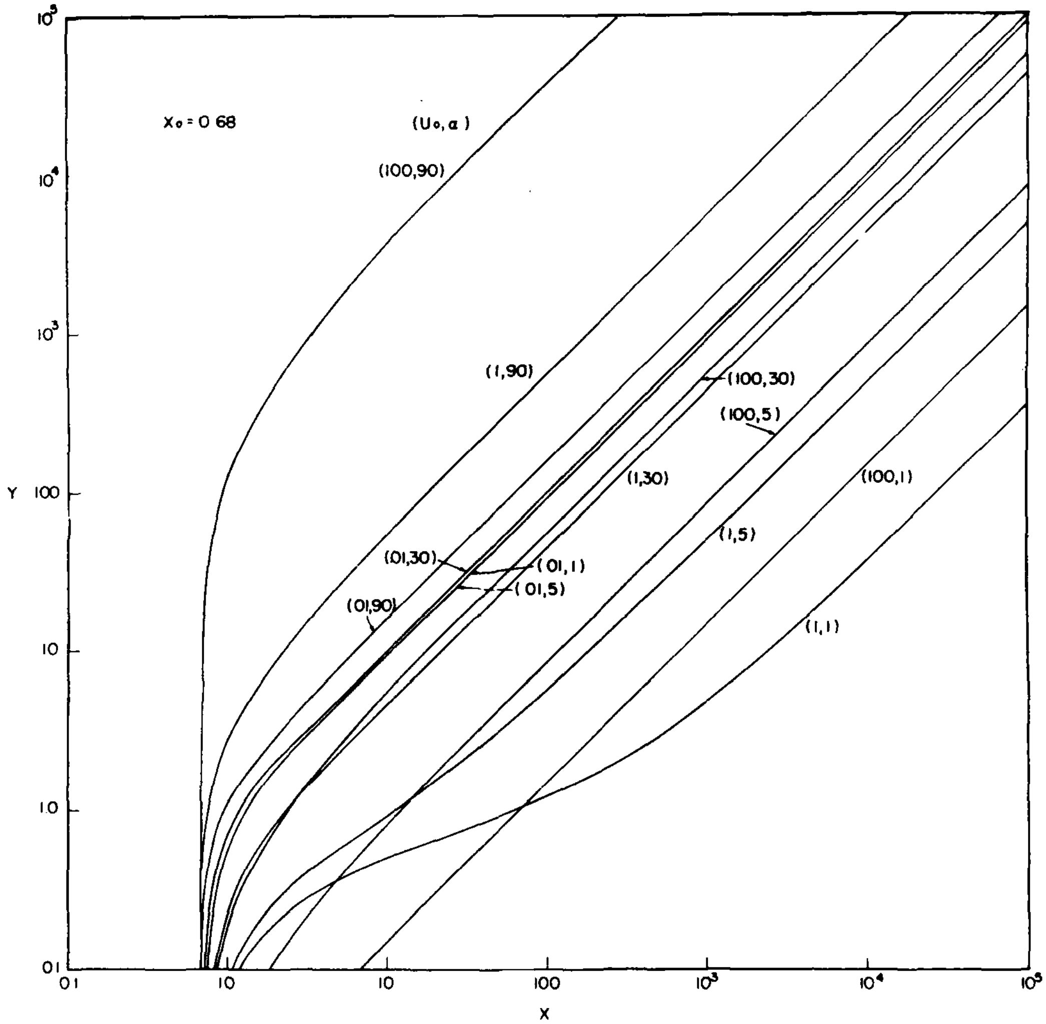

Figure 3. Typical Trajectories for $y_o = 0$, $x_o = 0.68$. The numbers in parenthesis are emission energy, U_o, and emission angle, α

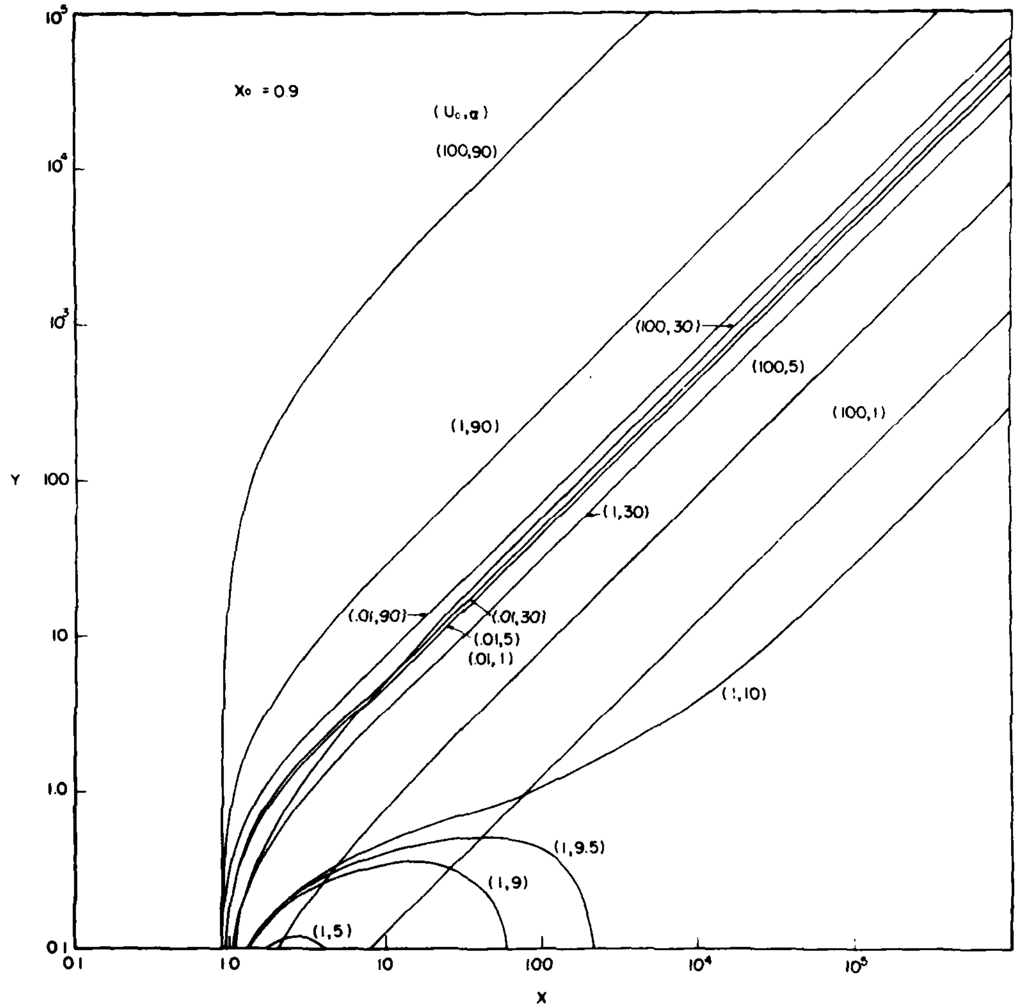

Figure 4. Typical Trajectories for $y_o = 0$, $x_o = 0.90$. The numbers in parenthesis are emission energy, U_o, and emission angle, α

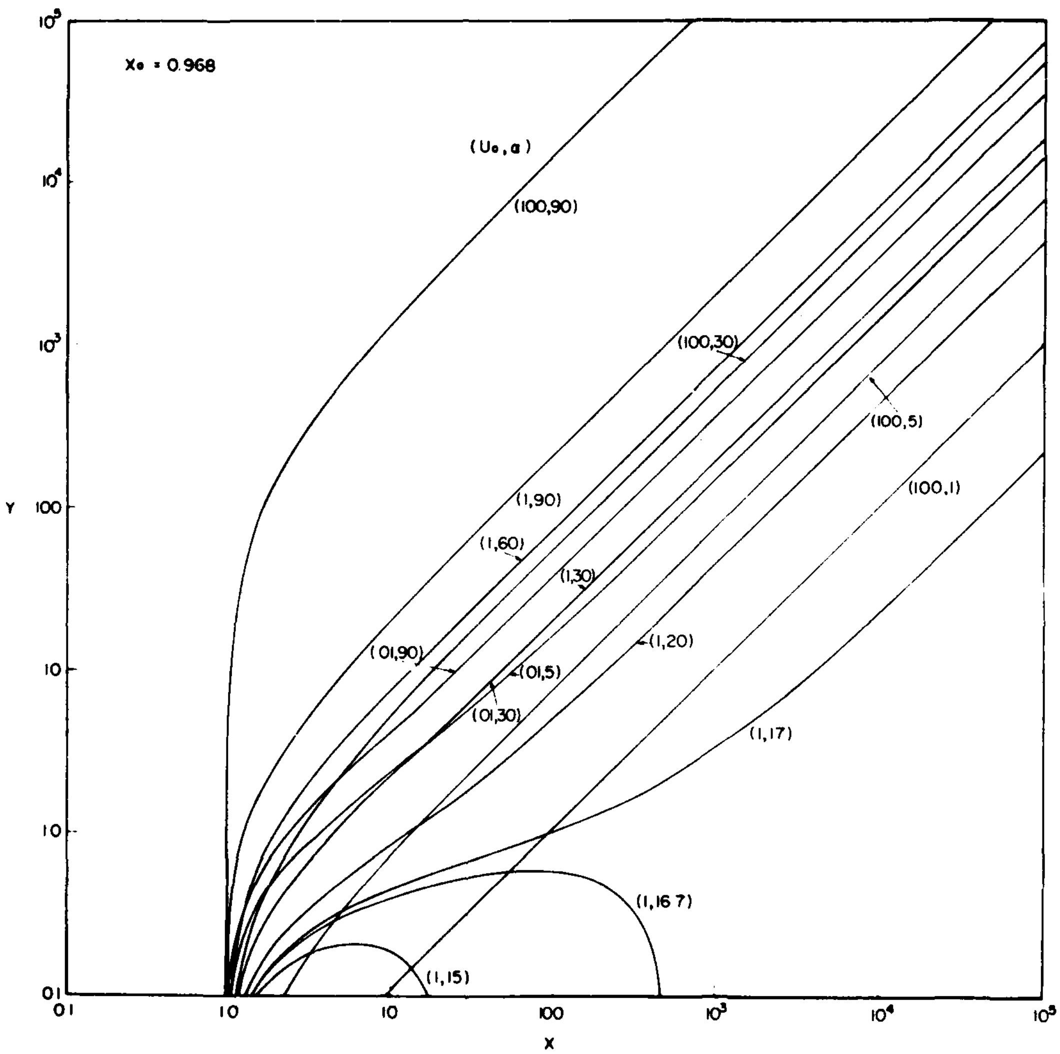

Figure 5. Typical Trajectories for $y_o = 0$, $x_o = 0.968$. The numbers in parenthesis are emission energy, U_o, and emission angle, α

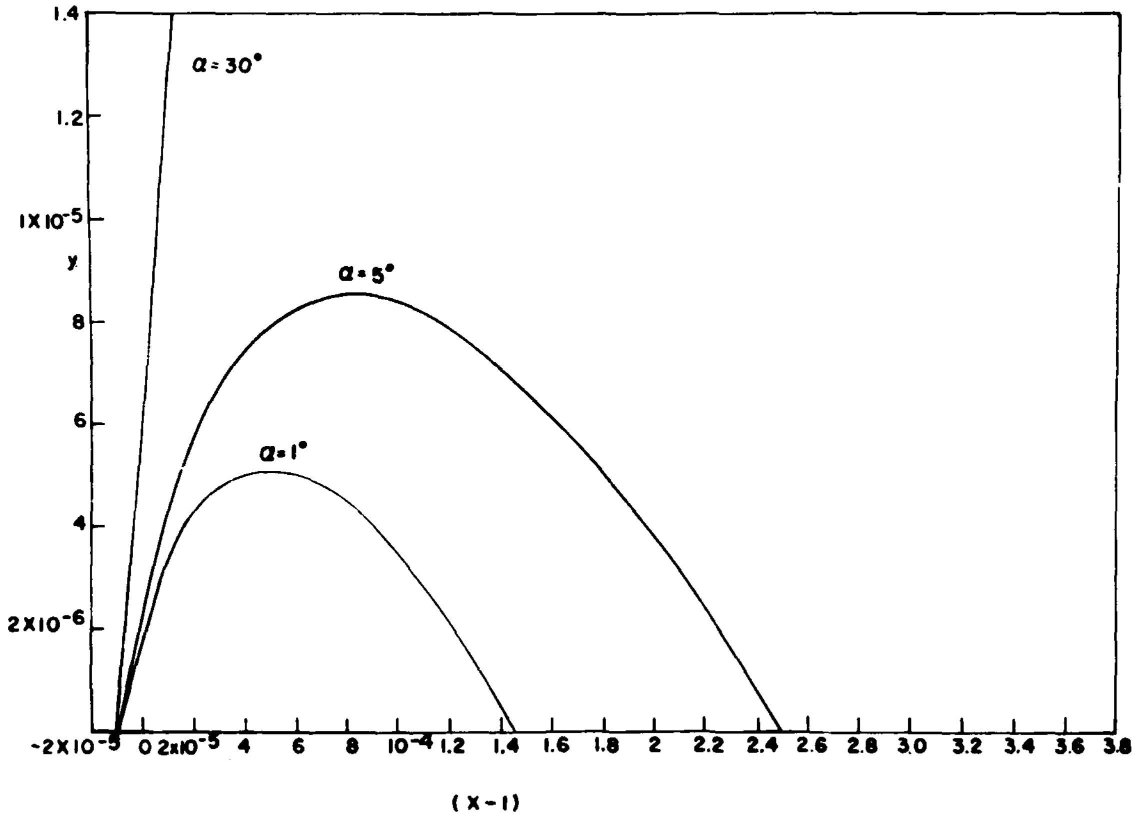

Figure 6. Typical Trajectories for Emission From Immediate Vicinity of Strip Edge Showing Similarity in Form to Those Obtained by Shima, Kalman, and Carini

4. THE LIMIT ALPHA APPROACHES ZERO

It is clear from the nature of the problem that if initial conditions $y_o > 0$, $\alpha > 0$ are chosen (that is, the particle is fired down on the outside electrode) it will always be possible to have the particle reach the plane of the electrode. Hence, it should not be a cause for surprise that solutions show a unique behavior for $y_o = 0$ and $\alpha \to 0$. Although values of α plotted in Figure 7 are only accurate to within

± 0.5 degree, the apparent peculiarities which arise for small angles of ejection were investigated. These questions are more of theoretical than of practical interest, since from an experimental viewpoint there is no possibility of discriminating between ejection angles of 0 and 1 degree. The results of this investigation are presented in the following table. Here, the numbers given are the x-coordinate for which the particle returns to y = 0. All values are for α = 0.0 degree.

U_o	70	80	90	100
x_o				
0.4	8.5	8.2	8.6	8.1
0.5	5.8	5.8	5.6	5.4
0.6	4.0	4.0	4.0	3.9
0.7	2.86	2.8	2.9	2.8

For identical values of U_o and x_o, but with α = 1.0 degree the particle escapes for all U_o, x_o. Thus, there is a radical difference in behavior between initial conditions having α = 0.0 and α = 1.0.

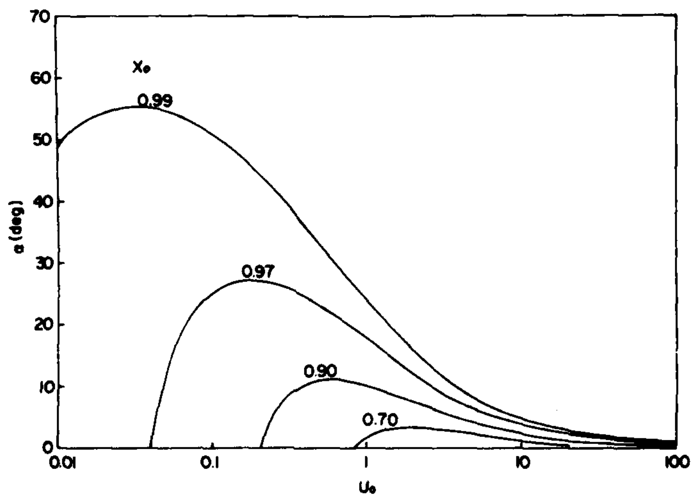

Figure 7. Separation Curves for Strip Problem. For any value (U_o, α) falling below a given curve, the particle is returned to the plane of emission; for any value (U_o, α) falling above a given curve, the particle escapes

5. ASYMPTOTIC PROPERTIES

If, in Eqs. (4) and (5) we set $\rho^2 = x^2 + y^2$ and allow x, y $\to \infty$ we obtain

$$\lim_{x,\, y \to \infty} \frac{d^2 x}{d\tau^2} \sim \lim_{x,\, y \to \infty} \frac{d^2 y}{d\tau^2} \sim 1/\rho^2 \; . \tag{9}$$

On the other hand, if in Eqs. (6) and (7) we let $\rho^2 = \xi^2 + \zeta^2$ and allow $\xi, \zeta \to \infty$ we have

$$\lim_{\xi,\, \zeta \to \infty} \frac{d^2 \xi}{d\tau^2} \sim \lim_{\xi,\, \zeta \to \infty} \frac{d^2 \zeta}{d\tau^2} \sim 1/\rho \; . \tag{10}$$

Thus it is seen immediately that the two sets of equations have different asymptotic properties.

To study the effects on trajectories of the asymptotic properties of Eqs. (4) and (5) we seek solutions of the type

$$x = \alpha\tau + \epsilon \tag{11}$$

$$y = \beta\tau + \eta \tag{12}$$

where

$$\lim_{\tau \to \infty} \frac{\epsilon}{\alpha\tau} = \lim_{\tau \to \infty} \frac{\eta}{\beta\tau} = 0 \; . \tag{13}$$

If we put Eqs. (11) and (12) into Eqs. (4) and (5), we have, to lowest order

$$\frac{d^2 \epsilon}{d\tau^2} = 4/\pi \, \frac{\alpha\beta}{(\alpha^2 + \beta^2)^2} \, \frac{1}{\tau^2} \tag{14}$$

$$\frac{d^2 \eta}{d\tau^2} = - \, 2/\pi \, \frac{\alpha^2 - \beta^2}{(\alpha^2 + \beta^2)^2} \, \frac{1}{\tau^2} \; . \tag{15}$$

Imposing the boundary conditions

$$\lim_{\tau \to \infty} \frac{d\epsilon}{d\tau} = \lim_{\tau \to \infty} \frac{d\eta}{d\tau} = 0$$

we find that ϵ, η, $\sim \ell n\,\tau$ and hence that solutions of the form Eqs. (11) and (12), approaching the limits of Eq. (13) are possible. This is consistent with the results of the numerical solutions.

If on the other hand, with x, y replaced by ξ, ζ respectively, we put Eqs. (11) and (12) into Eqs. (6) and (7) we find to lowest order

$$\frac{d^2\epsilon}{d\tau^2} = \frac{\beta}{\alpha^2+\beta^2}\ \frac{1}{\tau} \tag{16}$$

$$\frac{d^2\eta}{d\tau^2} = \frac{-\alpha}{\alpha^2+\beta^2}\ \frac{1}{\tau}\,. \tag{17}$$

Hence, regardless of what boundary conditions are imposed on $\frac{d\epsilon}{d\tau}$ and $\frac{d\eta}{d\tau}$ we obtain

$$\lim_{\tau\to\infty}\ \epsilon,\ \eta\ \sim \tau\ \ell n\,\tau\,,\ \text{and}$$

$$\lim_{\tau\to\infty} \frac{\epsilon}{\alpha\tau} = \lim_{\tau\to\infty} \frac{\eta}{\beta\tau} \rightarrow \ell n\,\tau \rightarrow \infty\,.$$

Thus there cannot exist solutions of the type of Eqs. (11) and (12) having limits of Eq. (13). This is consistent with the results of Shima, Kalman and Carini who find that all emitted particles return to the plane of origin.

6. COMMENTS ON THE CIRCULAR EMBEDDED ELECTRODE

From a modeling standpoint, it is clear that a circular electrode at one potential embedded in an infinite plane at a second potential will be a more realistic representation for most physical electrode systems than are the two models so far discussed. The price of this fidelity is increased labor. The solution of the boundary value problem of the circular electrode is available, but only as an infinite integral,

$$V = V_1 + \int_0^\infty J_1(\lambda)\, e^{-\lambda z}\, J_o(\lambda\rho)\ d\lambda\,. \tag{18}$$

Here, z, ρ are the usual cylindrical coordinates, J_o and J_1 are zeroth and first order Bessel functions, and a unit potential between inner and outer electrodes is again assumed. In the plane of the electrodes ($z = 0$) we have the infinite integral

$$\int_0^\infty J_1(\lambda)\, J_o(\rho\,\lambda)\, d\lambda = \begin{matrix} 1 & \rho^2 < 1 \\ 1/2 & \rho^2 = 1 \\ 0 & \rho^2 > 1 \end{matrix} \; . \qquad (19)$$

Along the axis of symmetry the integral is expressible as an elementary function

$$\int_0^\infty J_1(\lambda)\, e^{-\lambda z}\, d\lambda = 1 - \frac{z}{\sqrt{1+z^2}} \, . \qquad (20)$$

The general integral Eq. (18) has been tabulated[4] for a limited range of values of z and ρ. To compute trajectories however, it is necessary to follow particles out to large values of the independent variables. This might be accomplished by using a multipole expansion for large values; either a numerical solution or tabulation for small values; and matching at the boundary between, all in all, requiring a fair amount of labor.

The labor is probably not justified because of the following considerations. From Eq. (20), the asymptotic properties of the disk problem are available. We have

$$\lim_{z \to \infty} V \sim \frac{1}{z^2}$$

and for the field

$$\lim_{z \to \infty} E \sim \frac{1}{z^3} \, .$$

Thus the two half planes, the strip, and the circular disk form a sequence with respect to their asymptotic behavior. We expect they will also form a sequence with respect to the behavior of particles emitted from their surfaces. Since most initial conditions lead to particles which escape from the strip, this will be even more the case for the circular disk. We expect that only under very restricted initial conditions will particles emitted from a circular disk return to their plane of origin. The labor that would be required to solve for trajectories in this case is thus probably not justified.

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4. Eason, G., Noble, B., and Sneddon, I.N. (1955) Phil. Trans. Roy. Soc. London, 247:529-551.